Subject Name: **Heat & Mass Transfer**

Subject Code: **ME-6003**

Semester: **6th**

UNIT-II

Introduction

Heat transfer between a solid surface and a moving fluid is governed by the Newton's cooling law:

Many times, when the first option is not in our control and the second option (i.e. increasing) is already stretched to its limit, we are left with the only alternative of increasing the effective surface area by using fins or extended surfaces.

Fins are protrusions from the base surface into the cooling fluid, so that the extra surface of the protrusions is also in contact with the fluid.

Most of you have encountered cooling fins on air-cooled engines (motorcycles, portable generators, etc.), electronic equipment (CPUs), automobile radiators, air conditioning equipment (condensers) and elsewhere

Steady Flow of Heat along a Rod (Governing Differential Equation)

Consider a straight rectangular or pin fin protruding from a wall surface (figure 1 a, figure 1 b).

The characteristic dimensions of the fin are its length L, constant cross-sectional area and the circumferential parameter P.

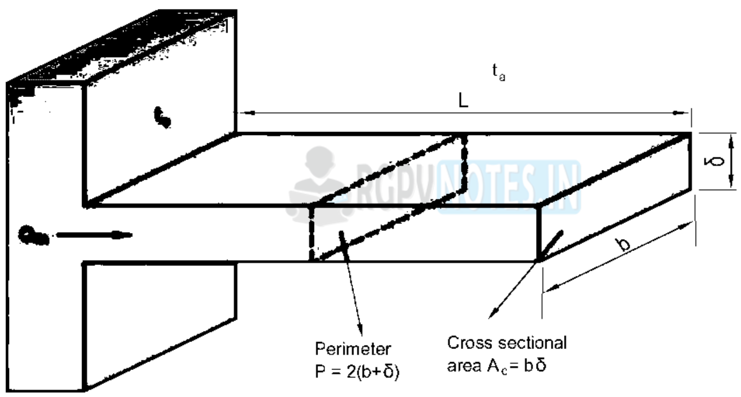

Fig. 1a Schematic diagram of a rectangular fin protruding from a wall

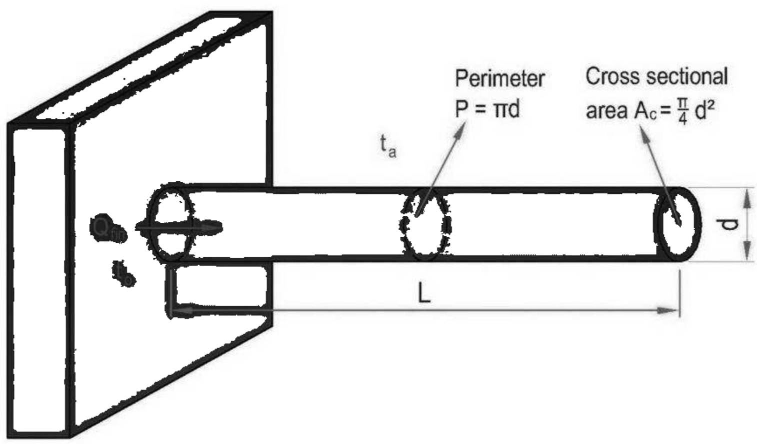

Fig. 1b Schematic diagram of a pin fin protruding from a wall

Analysis of heat flow from the finned surface is made with the following assumptions:

i Thickness of the fin is small compared with the length and width; temperature gradients over the cross-section are neglected and heat conduction treated one dimensional.

ii Homogeneous and isotropic fin material; the thermal conductivity k of the fin material is constant.

iii Uniform heat transfer coefficient h over the entire fin surface.

iv No heat generation within the fin itself.

v Joint between the fin and the heated wall offers no bond resistance; temperature at root or base of the fin is uniform and equal to temperature of the wall.

vi Negligible radiation exchange with the surroundings; radiation effects, if any, are considered as included in the convection coefficient h.
Vii Steady state heat dissipation.

Heat from the heated wall is conducted through the fin and convected from the sides of the fin to the surroundings.

Consider infinitesimal element of the fin of thickness dx at a distance x from base wall as shown in figure 2.

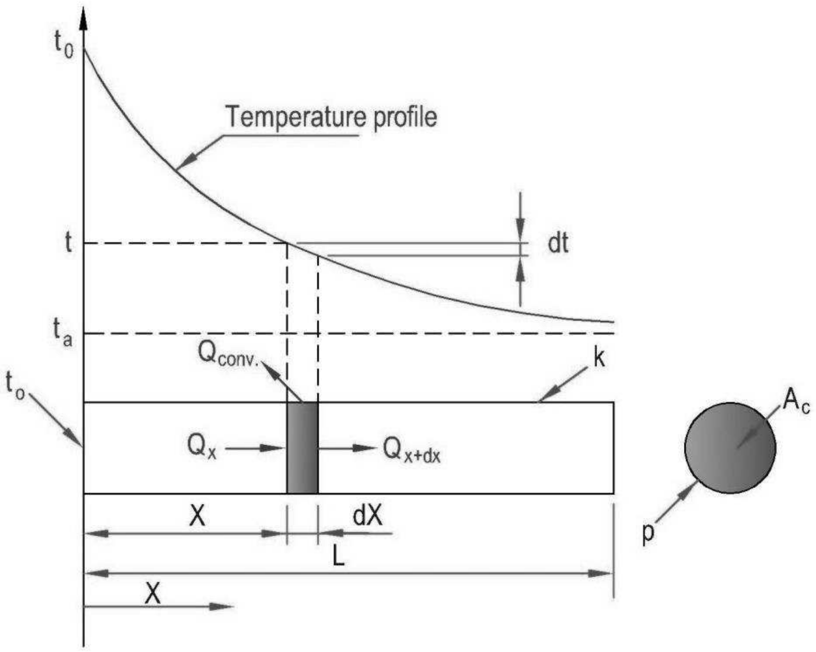

Fig. 2 Schematic diagram of a pin fin protruding from a wall

The general solution of this linear homogeneous second order differential equation is of the form

The constant and are to be determined with the aid of relevant boundary conditions. We will treat the following four cases:

i Heat dissipation from an infinitely long fin

ii Heat dissipation from a fin insulated at the tip

iii Heat dissipation from a fin losing heat at the tip

Heat Dissipation from an Infinitely Long Fin

Governing differential equation for the temperature distribution along the length of the fin is given as,

The relevant boundary conditions are

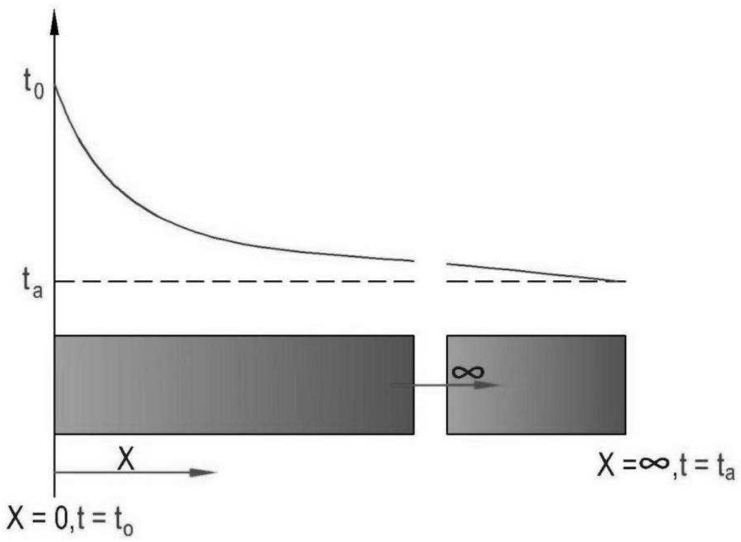

Fig. 3 Temperature distribution along the infinite long fin

Temperature at the base of fin equals the temperature of the surface to which the fin is attached.
In terms of excess temperature or

Substitution of this boundary condition in equation gives:

Temperature at the end of an infinitely long fin equals that of the surroundings.
Substitution of this boundary condition in equation gives:

Heat transfer from fin

Heat transfer to the fin at base of the fin must equal to the heat transfer from the surface of the fin by convection. Heat transfer to the fin at base is given as

From the expression for the temperature distribution

The temperature distribution would suggest that the temperature drops towards the tip of the fin.

Hence area near the fin tip is not utilized to the extent as the lateral area near the base. Obviously an increase in length beyond certain point has little effect on heat transfer.

So it is better to use tapered fin as it has more lateral area near the base where the difference in temperature is high.

Ingen-Hausz Experiment

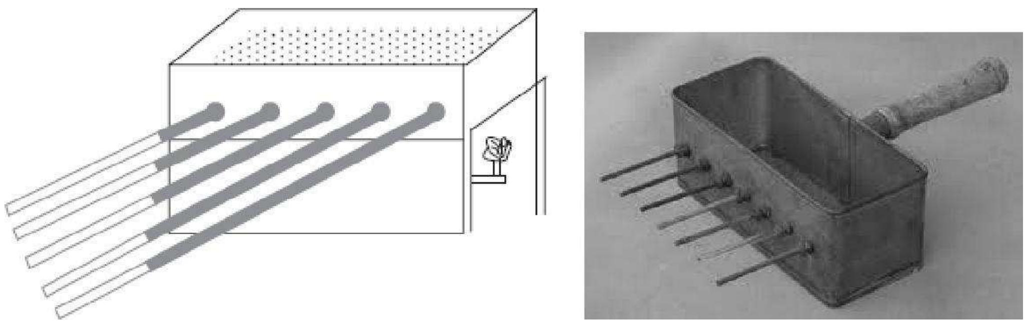

Fig. 4 Setup of Ingen-Hausz's Experiment

Heat flow rates through solids can be compared by having an arrangement consisting essentially of a box to which rods of different materials are attached (Ingen-Hausz experiment).

The rods are of same length and area of cross-section (same size and shape); their outer surfaces are electroplated with the same material and are equally polished.

Thus, the thermal conductivity of the material of the rod is directly proportional to the square of the length up to which the wax melts on the rod.

Heat Dissipation from a Fin Insulated At the Tip

The fin is of any finite length with the end insulated and so no heat is transferred from the tip.

Therefore, the relevant boundary conditions are:

Temperature at the base of fin equals the temperature of the surface to which the fin is attached.

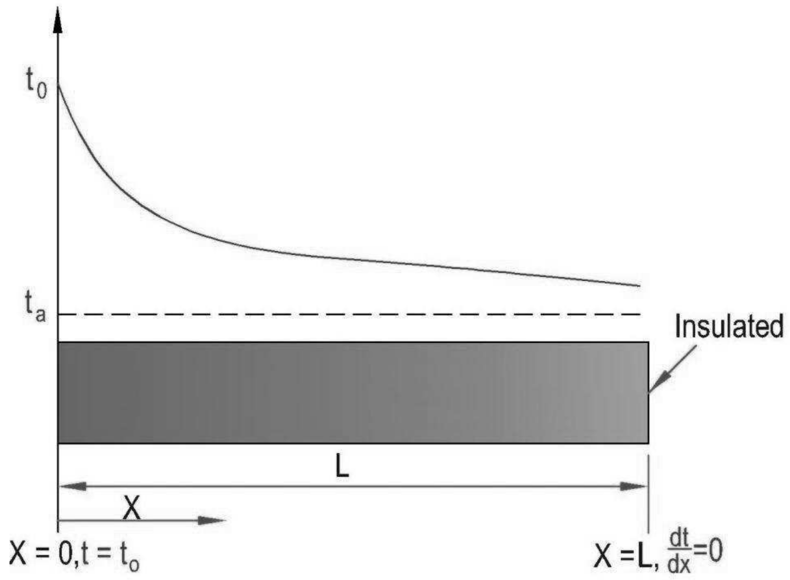

Fig. 5 Heat dissipation from a fin insulated at the tip

Heat Dissipation from a Fin Losing Heat at the Tip

The fin tips, in practice, are exposed to the surroundings. So heat may be transferred by convection from the fin tip.

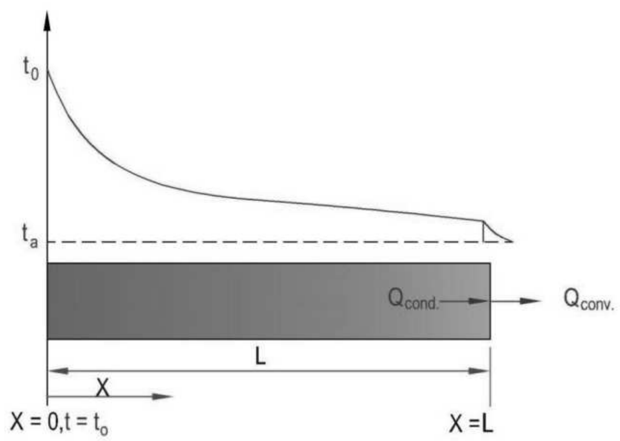

Fig. 6 Heat dissipation from fin losing heat at the tip

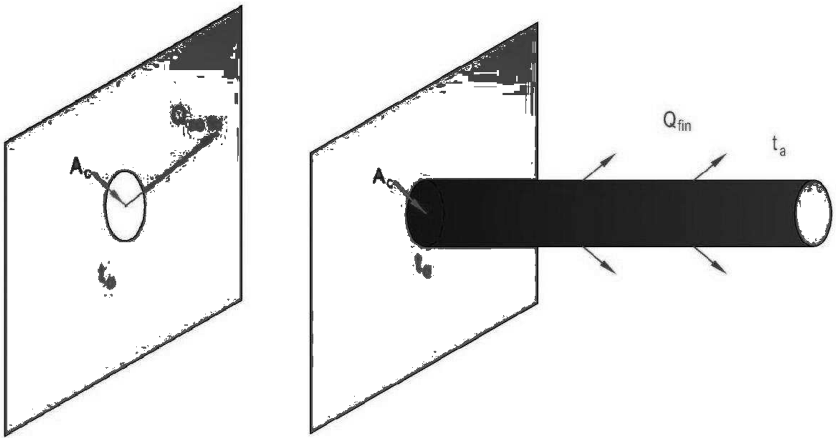

Fig. 7 Heat dissipation with and without fin

Following conclusions are given from the effectiveness of the fin

If the fin is used to improve heat dissipation from the surface, then the fin effectiveness must be greater than unity. That is,

But literature suggests that use of fins on surface is justified only if the ratio is greater than 5.

i To improve effectiveness of fin, fin should be made from high conductive material such as copper and aluminium alloys. Although copper is superior to aluminium regarding to the thermal conductivity, yet fins are generally made of aluminium because of their additional advantage related to lower cost and weight.

ii Effectiveness of fin can also be increased by increasing the ratio of perimeter to the cross sectional area. So it is better to use more thin fins of closer pitch than fewer thicker fins at longer pitch.

iii A high value of film coefficient has an adverse effect on effectiveness. So fins are used with the media with low film coefficient. Therefore, in liquid – gas heat exchanger, such as car radiator, fins are placed on gas side.

Relation between efficiency of fin and effectiveness of fin

An increase in the fin effectiveness can be obtained by extending the length of fin but that rapidly becomes a losing proposition in term of efficiency.

Thermometric Well

An arrangement which is used to measure the temperature of gas flowing through a pipeline.
A small tube called thermometric well is welded radially into the pipeline. The well is partially filled with some liquid and the thermometer is immersed into this liquid.

When the temperature of gas flowing through the pipe is higher than the ambient temperature, the heat flows from the hot gases towards the tube walls along the well. This may cause temperature at the bottom of well to become colder than the gas flowing around.

So the temperature indicated by the thermometer will not be the true temperature of the gas.

The error in the temperature measurement is estimated with the help of the theory of extended surfaces.

Fig. 8 Use of thermometric well

From the equation it is clear that diameter of the well does not have any effect on temperature measurement by the thermometer.

The error can be minimized by

i Lagging the tube so that conduction of heat along its length is arrested.

ii Increasing the value of parameter

Transient Heat Conduction

Introduction

In the preceding chapter, we considered heat conduction under steady conditions, for which the temperature of a body at any point does not change with time. This certainly simplified the analysis.

But before steady-state conditions are reached, sometime must elapse when a solid body is suddenly subjected to a change in environment. During this transient period the temperature changes and the analysis must take into account changes in the internal energy.

This study is a little more complicated due to the introduction of another variable namely time to the parameters affecting conduction. This means that temperature is not only a function of location, as in the steady state heat conduction, but also a function of time.

Transient heat flow is of great practical importance in industrial heating and cooling, some of the applications are given as follow

i Heating or cooling of metal billets;

ii Cooling of I.C. engine cylinder;

iii Cooling and freezing of food;

iv Brick burning and vulcanization of rubber;

v Starting and stopping of various heat exchanger units in power plant.

Change in temperature during unsteady state may follow a periodic or a non-periodic variation.

Periodic variation

The temperature changes in repeated cycles and the conditions get repeated after some fixed time interval. Some examples of periodic variation are given follow

i Variation of temperature of a building during a full day period of 24 hour

ii Temperature variation in surface of earth during a period of 24 hour

iii Heat processing of regenerators whose packing are alternately heated by flue gases and cooled by air

Non-periodic variation

The temperature changes as some non-linear function of time. This variation is neither according to any definite pattern nor is in repeated cycles. Examples are:

i Heating or cooling of an ingot in a furnace

ii Cooling of bars, blanks and metal billets in steel works

Transient Conduction in Solids with Infinite Thermal Conductivity (Lumped Parameter Analysis)

Even though no materials in nature have an infinite thermal conductivity, many transient heat flow problems can be readily solved with acceptable accuracy by assuming that the internal conductive resistance of the system is so small that the temperature within the system is substantially uniform at any instant.

This simplification is justified when the external thermal resistance (Convection resistance) between the surface of the system and the surrounding medium is so large compared to the internal thermal resistance (Conduction resistance) of the system that it controls the heat transfer process.

Consider a small hot copper ball coming out of an oven (Figure 4–1). Measurements indicate that the temperature of the copper ball changes with time, but it does not change much with position at any given time due to large thermal conductivity.

Thus the temperature of the ball remains uniform at all times.

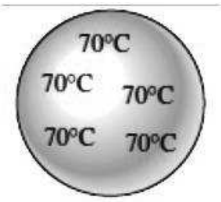

Fig. 9 Temperature distribution throughout the copper ball

Consider a body of arbitrary shape of mass m, volume V, surface area, density, and specific heat initially at a uniform temperature.

Following points can be made from the above equations:

1. The body temperature falls or rises exponentially with time and the rate depends on the parameter. Theoretically the body takes infinite time to approach the temperature of surroundings and thus attain the steady state conditions. However the difference between and becomes extremely small after a short time and beyond that period the body temperature becomes practically equal to the ambient temperature. The change in temperature of a body with respect to time is shown in figure 11 for both cases (Heating and cooling)

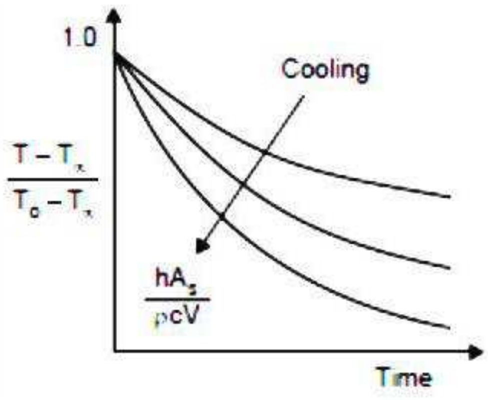

Fig. 10 Change in temperature of body with respect to time

2. The quantity has the dimensions of time and is called the thermal time constant. Its value is indicative of the rate of response of a system to a sudden change in the environmental temperature; how fast body will respond to a change in the environmental temperature. It should be as small as possible for fast response of the system to change in environmental temperature.

Exponential term can be arranged in dimensionless term as follow:

Criteria for Lumped System Analysis

Biot number is used to check the applicability of lumped parameter analysis. If Biot number is less than 0.1, it has been proved that this model can be used without appreciable error.

The lumped parameter solution for transient conduction can be conveniently stated as

Time Constant and Response of a Thermocouple

A Thermocouple is a sensor used to measure temperature. A thermocouple is comprised of at least two metals joined together to form two junctions.

One is connected to the body whose temperature is to be measured; this is the hot or measuring junction. The other junction is connected to a body of known temperature; this is the cold or reference junction.

Therefore the thermocouple measures unknown temperature of the body with reference to the known temperature of the other body.

Measurement of temperature by a thermocouple is an important application of the lumped parameter analysis.

The response of a thermocouple is defined as the time required for the thermocouple to reach the source temperature when it is exposed to it.

Referring to the lumped-parameter solution for transient heat conduction;

The sensitivity of the thermocouple is defined as the time required by the thermocouple to reach 63.2% of its steady state value. According to definition of sensitivity

The time constant represents the speed of response, i.e., how fast the thermocouple tends to reach the steady state value. A large time constant corresponds to a slow system response, and a small time constant represent a fast response. A low value of time constant can be achieved for a thermocouple by

i Decreasing light metals the wire diameter

ii Using light metals of low density and low specific heat

iii Increasing the heat transfer coefficient

Depending upon the type of fluid used, the response times for different sizes and materials of thermocouple wires usually lie between 0.04 to 2.5 seconds.

Note: - Once the time constant is measured, we have to wait for the time to measure the temperature within 63.2% of accuracy.

Transient Heat Conduction in Solids with Finite Conduction and Convective Resistance ($0 < B_i < 100$)

In the lumped parameter analysis we assume that conductivity of the material is infinite or variation of temperature within the body is negligible. But sometimes there may be variation of temperature with time and position.
Consider a plane wall of thickness 2L, a long cylinder of radius r_o, and a sphere of radius r_o initially at a uniform temperature T_i, as shown in figure 12.

Note that all three cases possess geometric and thermal symmetry: the plane wall is symmetry about its centre plane (x = 0), the cylinder is symmetry about its centreline (r = 0), and the sphere is symmetry about its centre point (r = 0).

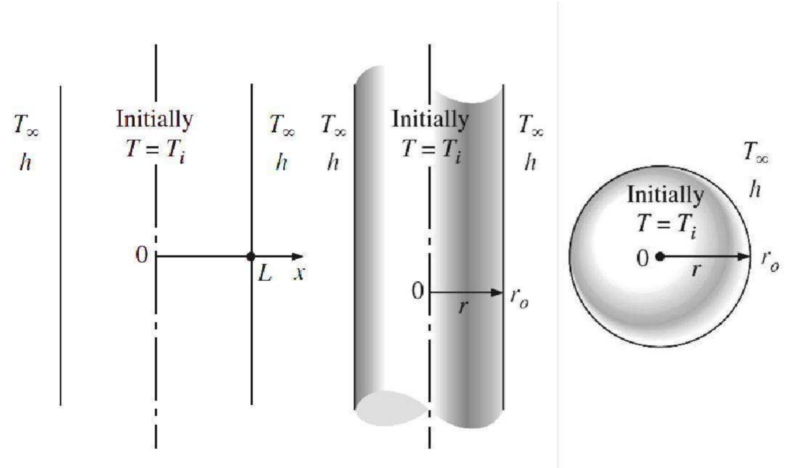

Fig. 11 Transient heat conduction in large wall, cylinder and sphere

Temperature profile of plane wall

The variation of temperature profile with respect to time in plane wall is shown in figure 13.

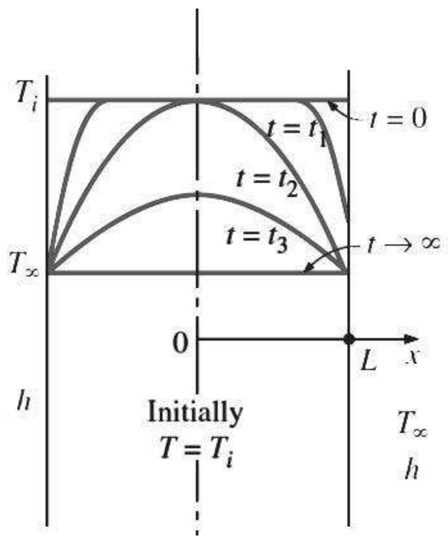

Fig. 12 Transient heat conduction in large wall, cylinder and sphere

When the wall is first exposed to the surrounding medium the entire wall is at its initial temperature .

But the wall temperature at the surface starts to drop as a result of heat transfer from the wall to the surrounding medium. This creates a temperature gradient in the wall.
The temperature profile within the wall remains symmetric at all times about the centre plane. The temperature profile gets flatter and flatter as times passes as a result of heat transfer and finally becomes uniform at. The controlling differential equation for the transient heat conduction is:

The solution of the controlling differential equation in conjunction with initial boundary conditions would give an expression for temperature variation both with time and position.

The temperatures at other locations are worked out by multiplying the mid-plane temperature by correction factors read from correction charts.

The Heisler charts are extensively used to determine the temperature distribution and heat flow rate when both conduction and convection resistances are almost of equal importance.

